# Local resilience of an almost spanning $k$-cycle in sparse random graphs

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**Abstract**

Pósa and Seymour conjectured that for any $k \geq 2$, a graph with minimum degree at least $kn/(k+1)$ contains the $k$-th power of a Hamilton cycle, namely, a Hamilton cycle where additionally between every pair of vertices at distance at most $k$, there is an edge. Only much later, after appearance of tools such as Szemerédi's Regularity Lemma and the Blow-up Lemma, Komlós, Sárközy, and Szemerédi confirmed the Pósa-Seymour conjecture for large graphs.

We extend this result to a sparse setting by showing that for all $k \geq 2$ and $\alpha, \varepsilon > 0$, there exists a $C > 0$, such that for $p \geq C(\log n/n)^{1/k}$, any subgraph of a random graph $G_{n,p}$ with minimum degree $(k/(k+1) + \alpha)np$, w.h.p. contains the $k$-th power of a cycle on $(1-\varepsilon)n$ vertices, thus improving upon the recent results of Noever and Steger for $k = 2$, as well as Allen, Böttcher, Ehrenmüler, and Taraz for $k \geq 3$.

Our result is almost optimal for the following reasons. The constant $k/(k+1)$ in the bound on the minimum degree cannot be improved. When $p \ll n^{-1/k}$, $G_{n,p}$ does not contain the $k$-th power of a long cycle w.h.p. Finally, just by deleting an $\varepsilon$-fraction of the edges touching each vertex, it is easy for an adversary to make sure that some vertices are not contained in the $k$-th power of a cycle, which shows that one cannot hope for an improvement of the result where $k$-th power of a Hamilton cycle is obtained instead of a cycle on $(1-\varepsilon)n$ vertices.

This is joint work with Angelika Steger and Nemanja Škorić.