Packing nearly optimal $R(3,t)$ graphs

He Guo

Georgia Institute of Technology

In a celebrated paper from 1995, Kim proved the Ramsey bound $R(3,t) \geq ct^2/\log t$ by constructing an n-vertex graph that is triangle-free and has independence number at most $C\sqrt{n \log n}$. We extend this result, which is best possible up to the value of the constants, by approximately decomposing the complete graph K_n into a packing of such nearly optimal $R(3,t)$ graphs. More precisely, for any $\epsilon > 0$ we find an edge-disjoint collection $(G_i)_i$ of n-vertex graphs $G_i \subseteq K_n$ such that (a) each G_i is triangle-free and has independence number at most $C_\epsilon\sqrt{n \log n}$, and (b) the union of all the G_i contains at least $(1-\epsilon)\binom{n}{2}$ edges. Our algorithmic proof proceeds by sequentially choosing the graphs G_i via a semi-random (i.e., Rödl nibble type) variation of the triangle-free process. As an application we prove a conjecture of Fox, Grinsh, Liebenau, Person and Szabo in Ramsey theory (concerning r-Ramsey-minimal graphs for K_3).

Joint work with Lutz Warnke.