Minimum number of edges that occur in odd cycles

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Abstract

For any fixed $k > 1$ every graph on $n \geq 4k$ vertices without C_{2k+1} has at most $\lfloor n^2/4 \rfloor$ edges. In 1992, Erdos, Faudree and Rousseau conjectured that if a graph has at least $\lfloor n^2/4 \rfloor + 1$ edges, then already at least $2n^2/9 + O(n)$ edges occur in a copy of C_{2k+1}.

In the talk we will disprove this conjecture for $k = 2$ and prove the correct bound, i.e., that any n-vertex graph with at least $\lfloor n^2/4 \rfloor + 1$ edges has at least $(2 + \sqrt{2})n^2/16 - O(n^{15/8})$ edges that occur in C_5.

Next, for every $k > 2$ we will prove that the conjecture is true, i.e., that any n-vertex graph with at least $\lfloor n^2/4 \rfloor + 1$ edges has at least $\lfloor \frac{n^2}{4} \rfloor + 1 - \lfloor \frac{n+4}{6} \rfloor \lfloor \frac{n+1}{6} \rfloor = 2n^2/9 - O(n)$ edges that occur in C_{2k+1}.

This is a joint work with Ping Hu and Jan Volec.